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New Connections Between String Theories

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ABSTRACT

We consider the $R \to 0$ limit of toroidal compactification in various string theories. This leads to new connections between seemingly different string theories: IIA and IIB, open and closed, oriented and unoriented. We also find two new extended objects which can couple consistently to strings: the Dirichlet-brane and the orientifold plane.

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1 INTRODUCTION

One of the remarkable features of string theory is duality [1-4]. Consider the closed oriented bosonic string on the spacetime $M^{d_c-1} \times T^1$, where one dimension has been periodically compactified with circumference $2\pi R$. This is physically equivalent to the same theory compactified with radius $R' = \alpha'/R$, in both its spectrum and interactions. This has the striking consequence that the $R \to 0$ limit is the same as $R \to \infty$. States with non-zero momentum in the compact dimension became infinitely heavy as $R \to 0$, but states which wind around the compact dimension become light, forming a continuum in the limit. This is one piece of evidence for the idea that there is a minimum length in string theory: the self-dual radius $R = \sqrt{\alpha'}$ is, physically, the minimum possible radius. These same results hold for the heterotic string, for more than one toroidally compactified dimension, and for some more general compactifications.

In this letter we consider toroidal compactification of various string theories which are *not* self-dual. The $R \to 0$ limit of such a compactification gives the $R \to \infty$ limit of a different string theory. In this way we discover new connections between string theories, parallel to the connection between the $SO(32)$ and $E(8) \times E(8)$ heterotic string theories [5]. These new connections are between IIA and IIB theories [3], open and closed theories, and unoriented and oriented theories. We also find new extended objects in string theories: the D-brane and the orientifold.

In the following sections we consider closed oriented, open oriented, closed unoriented, and open unoriented string theories. In a final section we summarize and offer some speculations.

2 Closed Oriented Strings

We review the bosonic case first. It is sufficient for our purposes to consider toroidal compactification corresponding to several periodic dimensions, $x^i \sim x^i + 2\pi R_i$. (We use $i, j, \ldots$ for compact dimensions, $m, n, \ldots$ for noncompact, and $\mu, \nu \ldots$ for all). The holomorphic and antiholomorphic parts of X^μ are expanded

$$
\begin{aligned}
X^\mu_R(z) &= X^\mu_0 + i\,\frac{\alpha'}{2}\, p^\mu_R \ln z + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{z^{-n}}{n}\, \alpha^\mu_n \\
X^\mu_L(\bar z) &= \tilde X^\mu_0 + i\frac{\alpha'}{2}\, p^\mu_L \ln \bar z + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{\bar z^{-n}}{n}\, \tilde\alpha^\mu_n \ .
\end{aligned}
\tag{1}
$$

For the compactified dimensions, the eigenvalues of p_R and p_L are restricted to

$$
p^i_R = \frac{n_i}{R_i} + \frac{m_i R_i}{\alpha'} \qquad p^i_L = \frac{n_i}{R_i} - \frac{m_i R_i}{\alpha'} \qquad m_i,\ n_i \in \boldsymbol{Z} \ , \tag{2}
$$

while for a noncompact dimension $p^a_R = p^a_L$ is continuous. The mass squared is

$$
M^2 = p^i_R p^i_R + N = p^i_L p^i_L + \tilde N \ , \tag{3}
$$

where N and $\tilde N$ are the sums of the right-moving and left-moving oscillator levels. We see that the state $(\vec n, \vec m)$ at radii R_i has the same mass-squared

as $(\vec{m}, \vec{n})$ at the dual radii $R'_i = \alpha'/R_i$ [1]. Further, the interactions are the same. Defining

$$
\begin{aligned}
Y^\mu_R(z) &= X^\mu_R(z) \\
Y^\mu_L(\bar{z}) &= (-1)^{S^\mu} X^\mu_L(\bar{z}) \ ,
\end{aligned}
\tag{4}
$$

where $S^\mu = 0$ for a noncompact and 1 for a compact dimension, the vertex operators of the theory at radius R go over to the vertex operators of the dual states, and the Green functions of the Y^μ are equal to those of the X^μ [2].

For the heterotic string, and the IIA and IIB theories, the duality of the spectra follows in the same way; compactification does not affect the various fermionic fields. However, for an odd number of compact dimensions the $R_i \to 0$ limit of the IIA theory is physically inequivalent to the $R_i \to \infty$ limit[f1]. One way to see this is by considering the world-sheet changes $L^{\mu\nu}$ corresponding to spacetime Lorentz symmetry. We can specialize to a single compact dimension X^9. In the $R_9 \to \infty$ limit, one sees from eq. (2) that all states of finite mass satisfy

$$
R_9 = \infty : \quad p^\mu_R |\psi\rangle = p^\mu_L |\psi\rangle \tag{5}
$$

with the eigenvalues of $p^\mu_R = p^\mu_L$ taking continuous values. In the $R_9 \to 0$ limit, all states of finite mass satisfy

$$
R_9 = 0 : \qquad p^\mu_R |\psi\rangle = (-1)^{S^\mu} p^\mu_L |\psi\rangle \ , \tag{6}
$$

with $p^\mu_{R,L}$ taking continuous values. The world-sheet supercharge (in the Ramond-Ramond sector) is

$$T_F = p^\mu_R \psi^\mu_0 + p^\mu_L \tilde{\psi}^\mu_0 \ + \ \cdots \ , \tag{7}$$

independent of R_9, where we indicate explicitly only the zero mode terms. The Lorentz generators are therefore

$$\begin{aligned} R_9 = \infty : \quad L^{\mu\nu} &= X_0^{[\mu} p_R^{\nu]} + \frac{i}{2}\, \psi_0^{[\mu} \psi_0^{\nu]} + \tilde{X}_0^{[\mu} p_l^{\nu]} + \frac{i}{2}\, \tilde{\psi}_0^{[\mu} \tilde{\psi}_0^{\nu]} + \ldots \\ R_9 = 0 : \quad L^{\mu\nu} &= X_0^{[\mu} p_R^{\nu]} + \frac{i}{2} \psi_0^{[\mu} \psi_0^{\nu]} \\ &\quad + (-1)^{S^\mu + S^\nu} \left(\tilde{X}_0^{[\mu} p_L^{\nu]} + \frac{i}{2}\, \tilde{\psi}_0^{[\mu} \tilde{\psi}_0^{\nu]} \right) + \ldots \ . \end{aligned} \tag{8}$$

The form of $L^{\mu\nu}$ in the two limiting theories (there is no $L^{\mu\nu}$ for $0 < R_9 < \infty$, of course) is fixed by the requirement that it take physical states into physical states: the bosonic terms in (8) follow because $L^{\mu\nu}$ must respect the condition (5) or (6), and the fermionic terms then follow from the requirement that $L^{\mu\nu}$ commute with T_F.

The IIA and IIB theories can be distinguished by the Lorentz representation of the Ramond-Ramond ground state, in particular by the eigenvalue, ± 1, of

$$\Gamma = \prod_{k=0}^{4} \left(4 S^{2k,2k+1} \tilde{S}^{2k,2k+1} \right) \tag{9}$$

where $S^{\mu\nu}$, $\tilde{S}^{\mu\nu}$ are the right and left moving halves of the fermionic part of $L^{\mu\nu}$. We see that, while the fermionic state is R_9-independent, the operators $\tilde{S}^{89}$ at $R_9 = \infty$ and $R_9 = 0$ differ by a sign, and so the eigenvalue of Γ is

opposite in the two theories: one is IIA and the other is IIB. It therefore also follows that the interactions are different [3].

This equivalence is not surprising, since the 16 and $\overline{16}$ representations of SO(9,1) reduce to the same 16 of SO(8,1), but it is amusing that this connection is even simpler than that between the SO(32) and $E(8) \times E(8)$ heterotic strings [5].

3 Open Oriented Strings

It is clear that the open string spectrum is not self-dual. An open string has no winding number, so there are no states to become light as $R_i \rightarrow 0$. If we make k dimensions periodic in the bosonic open string theory, and take $R_i \rightarrow 0$, the open strings in the limiting theory propagate in $26-k$ spacetime dimensions, although their oscillations still span the full 26 dimensions. To see the nature of the dual theory, consider the open string boundary condition

$$\begin{aligned} 0 = \partial_n X^i &= \left(\frac{\partial z}{\partial \tau}\right) \partial X^i - \left(\frac{\partial \bar{z}}{\partial \tau}\right) \bar{\partial} X^i \\ &= \left(\frac{\partial z}{\partial \tau}\right) \partial Y^i + \left(\frac{\partial \bar{z}}{\partial \tau}\right) \bar{\partial} Y^i \\ &= \partial_\tau Y^i \ . \end{aligned} \tag{10}$$

In terms of the coordinate Y^i of the dual theory, eq. (4), we see that the usual open string Neumann boundary conditions have become Dirichlet conditions: the Y^i are constant along each boundary. Actually, the Y^i are the same on every boundary. We can see this for the two ends of an open string from the

open string mode expansion

$$
\begin{aligned}
X_R^\mu &= \frac{1}{2}X_0^\mu + i\alpha' p^\mu \ln z + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{z^{-n}}{n}\alpha_n^\mu \\
X_L^\mu &= \frac{1}{2}X_0^\mu + i\alpha' p^\mu \ln \bar{z} + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\bar{z}^{-n}}{n}\alpha_n^\mu
\end{aligned} \tag{11}
$$

where $p^i = n_i/R_i$. The positive real line is the right end of the string and the negative real line the left end, so the dual coordinate $Y^i = X_R^i - X_L^i$ satisfies

$$
\begin{aligned}
Y^i(\sigma=\pi) - Y^i(\sigma=0) &= 2\pi\alpha' p^i \\
&= 2\pi\alpha' n_i/R_i \\
&= 2\pi n_i R_i' \ .
\end{aligned} \tag{12}
$$

That is, $Y^i(\sigma=\pi)$ and $Y^i(\sigma=0)$ are identical points on the dual torus. To show this for any pair of boundaries, consider a contour C connecting them.

$$
\begin{aligned}
Y_{B_2}^i - Y_{B_1}^i &= \int_C d\sigma^a \partial_a Y^i \\
&= i\int_C d\sigma^a \epsilon_a{}^b \partial_b Y^i \\
&= \alpha' p_C^i = 2\pi\alpha' n_i/R_i = 2\pi n_i R_i' \\
&\simeq 0 \ .
\end{aligned} \tag{13}
$$

Here, p_C^μ is the spacetime momentum flowing across C, and again the two boundaries are at the same point of the dual torus.

We have found that, if k spatial dimensions are compactified, and $R_i \to 0$, we obtain a theory of open strings in which the end points are confined to

the $(26-k)$ plane $Y^i = 0$. Of course, any theory of open strings must also contain closed strings, and these still propagate in 26 dimensions in the dual theory. Since the closed string sector contains gravity, one would expect that the $Y^i = 0$ hyperplane cannot be rigid but must have dynamics. That is, some open string states propagating along the hyperplane can in fact be interpreted as collective motions of the hyperplane. On symmetry grounds we would expect these states to be the dual of the perpendicular $U(1)$ gauge boson, with vertex operator

$$\int_B ds\; A^i(Y^m)\partial_n Y^i \quad . \tag{14}$$

To verify this, we examine the effective low energy theory of the membrane and gravity. The invariant membrane action of lowest dimension is

$$S = T' \int d^{26-k}\sigma \,\det{}^{1/2} \left[\frac{\partial Y^\mu}{\partial \sigma^a} \frac{\partial Y^\nu}{\partial \sigma^b} G_{\mu\nu}(Y) \right] \quad . \tag{15}$$

We expand around flat spacetime and a flat membrane, using the membrane coordinates

$$\sigma^a = Y^a \ , \ \ a = 0, \ldots 25-k \quad . \tag{16}$$

Then

$$\begin{aligned} G_{\mu\nu}(Y) &= \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(Y) \\ Y^i(\sigma) &= \frac{1}{\sqrt{T'}} \phi^i(\sigma) \quad . \end{aligned} \tag{17}$$

The $h_{\mu\nu}$ and the membrane fluctuations ϕ^i are canonically normalized. Expanding the action (14), one finds a term of order $h\phi$, corresponding to a graviton hitting the membrane and making it vibrate:

$$\phi^j \; \text{- - - -} \cdot \text{———} \; h^a_i$$

$$= -2\kappa\sqrt{T'}k^a\delta^j{}_i \quad . \tag{18}$$

A direct string calculation, the disk with one dual gauge boson and one graviton, gives the form (18) and determines the membrane tension T' to be

$$\begin{aligned} T' &= \frac{1}{2\pi^2\alpha' g^2_{26-k}} \\ &= \frac{T}{\pi g^2_{26-k}} \quad . \end{aligned} \tag{19}$$

Here g_{26-k} is the open string coupling in the effective $26-k$ dimensional theory on the world-sheet, as defined by the coefficient of the open string tachyon vertex operator. Computation of other graphs, such as $0(\phi^4)$ (scattering of membrane vibrations) gives the same result [6].

A word about couplings: in the theory with k periodic dimensions, the $26-k$ dimensional gravitational and open string couplings are related to the 26 dimensional couplings by

$$\begin{aligned} \kappa_{26-k} &= \kappa_{26}V_k^{-1/2} \\ g_{26-k} &= g_{26}V_k^{-1/2} \end{aligned} \tag{20}$$

where $V_k = \Pi_i(2\pi R_i)$. The gravitational coupling in the dual theory is

$$\begin{aligned} \kappa'_{26} &= \kappa_{26-k}(V'_k)^{1/2} = \kappa_{26}\left(\frac{V'_k}{V_k}\right)^{1/2} \\ &= \kappa_{26}(4\pi^2\alpha')^{k/2}V_k^{-1} \end{aligned} \tag{21}$$

Thus, to take $V_k \to 0$ and obtain a finite coupling in the dual theory, κ_{26}/V_k must be held fixed. Using (20), (21), and the relation $g_{26}^2 = C\kappa_{26}$ gives[f2]

$$g_{26-k}^2 = \frac{C}{(4\pi^2\alpha')^{k/2}}\, \kappa'_{26} \quad .$$

Thus, the relation between the dual open and closed string couplings, g_{26-k} and κ'_{26}, is finite as $V_k \to 0$.

To summarize, the dual theory to a theory of open plus closed oriented strings is a theory of closed strings coupled to a new dynamic object, the "D-brane" (short for Dirichlet-brane). The perpendicular $U(1)$ gauge boson becomes the collective coordinate for motion of the D-brane. The remaining perpendicular gauge bosons, of $SU(N)$, do not appear to have any such collective interpretation. The extension of the low energy effective action (15) to the full set of massless fields (adding the dilaton, antisymmetric tensor, parallel $U(1)$ and the $SU(N)$ gauge bosons) is under study [6].

Open strings with Dirichlet boundary conditions have been considered at various times in the past. In [8], Dirichlet conditions on some coordinates were used to compactify the open string. A boundary at which all coordinates (space and time) satisfy Dirichlet conditions has been interpreted as an off-shell probe or as a shift of the string vacuum[9], while open strings with one or both ends fixed on an orbifold point arise in orbifold compactification of open string theories [10]. However, as far as we are aware, the present work is the first interpretation of a Dirichlet hyperplane as an actual dynamical object, which can couple in a consistent way to closed strings. Note that

the only consistent theory of open oriented strings is the bosonic theory, so we have only found a bosonic D-brane; the supersymmetric case will be discussed in Section 5.

4 Closed Unoriented Strings

The twist operator Ω interchanges untilded and tilded operators in the mode expansion (1), so that

$$\Omega X_R^\mu(z)\Omega^{-1} = X_L^\mu(\bar{z}) \quad . \tag{22}$$

Thus,

$$\Omega\alpha_{-I}\tilde{\alpha}_{-J}|x\rangle = \alpha_{-J}\tilde{\alpha}_{-I}|x\rangle \tag{23}$$

where $\alpha_{-I}, \tilde{\alpha}_{-J}$ stand for arbitrary products of α and $\tilde{\alpha}$'s, and $|x\rangle$ is the closed string ground state in an eigenstate of $x^\mu = X_0^\mu + \tilde{X}_0^\mu$. The unoriented theory is defined by projecting

$$\Omega|\psi\rangle = +|\psi\rangle \quad . \tag{24}$$

From (23) we see that this restricts the string state to even tensor structures, $(\alpha_{-I}\tilde{\alpha}_{-J} + \alpha_{-J}\tilde{\alpha}_{-I})\,|x\rangle$.

In terms of the dual coordinate,

$$\Omega Y_R^\mu(z)\Omega^{-1} = (-1)^{S^\mu} Y_L^\mu(\bar{z}) \quad . \tag{25}$$

Thus,

$$\Omega\alpha^Y_{-I}\tilde{\alpha}^Y_{-J}|y\rangle = (-1)^{\#}\alpha^Y_{-J}\tilde{\alpha}^Y_{-I}|y'\rangle \quad , \tag{26}$$

where $\#$ counts the number of i indices among the I and J, and $(y^\mu)' = (-1)^{S^\mu} y^\mu$. Thus, states in the unoriented theory are of the form

$$\alpha^Y_{-I}\tilde{\alpha}^Y_{-J}|y\rangle + (-1)^{\#}\alpha^Y_{-J}\tilde{\alpha}^Y_{-I}|y'\rangle \ . \tag{27}$$

We see that the Ω projection puts no restriction on the tensor structure (away from $y^i = 0$), but relates it to the flipped tensor structure at a *different* point. Away from the $y^i = 0$ hyperplane, we therefore have the spectrum of the *oriented* closed string. The strings propagate, not in ordinary spacetime, but on an orbifold, since the fields at y' are determined in terms of those at y. This is not exactly the same as the orbifolds considered in ref.[11], because the Z_2 twist Ω is the product of a Z_2 symmetry of the dual spacetime and a Z_2 symmetry, orientation reversal, on the world-sheet. We therefore refer to the space as an "orientifold." Away from the orientifold (hyper)plane $y_i = 0$, the spectrum and interactions are locally indistinguishable from the closed oriented string; near the plane, unoriented topologies contribute.

5 Open Unoriented Strings

From the results of the previous two sections, it is clear that the dual to a theory of open plus closed unoriented strings is a closed oriented theory with both a D-brane and an orientifold plane. The question is, are these a single object, with the open string ends fixed on the orientifold plane, or are they two independent objects? It is easy to see that the former is the case. The

twist (25) of the dual coordinate $\left(Y^\mu(z,\bar{z}) = X_R^\mu(z) + (-1)^{S^\mu} X_L^\mu(\bar{z})\right)$,

$$\Omega Y^\mu(z,\bar{z})\Omega^{-1} = (-1)^{S^\mu} Y^\mu(\bar{z},z) \quad , \tag{28}$$

can be decomposed into a world-sheet twist and a spacetime operation:

$$Y^\mu(z,\bar{z}) \to Y^\mu(\bar{z},z) \to (-1)^{S^\mu} Y^\mu(\bar{z},z) \quad . \tag{29}$$

The orientifold plane is the fixed line of the spacetime symmetry, $Y^i = 0$. From the mode expansion (11), this is also the plane on which the endpoints lie[f3]. Further, the projection $\Omega = +1$ removes the open string state $\alpha^i_{-1}|0\rangle$, which would correspond to collective motion of the D-brane awayfrom the orientifold plane.

Note that we have not been able to construct a super-D-brane. Since the only consistent open superstring theory is unoriented, the would-be super-D-brane is hidden in the orientifold plane. If we took the dual to a theory of oriented open superstrings, we would obtain a super-D-brane which was consistent to lowest (disk) order, but the anomaly of the original theory would presumably appear at next order. Similar difficulties in producing supersymmetric Dirichlet boundaries have been found by M.B. Green (private communication). Anomaly considerations imply that string boundaries and crosscaps should be found together; hence the D-brane lies on the orientifold plane.

6 Conclusions

We have found new connections between string theories: starting from any string theory, open or closed, oriented or unoriented, one can obtain via compactification a theory of closed oriented strings propagating in a spacetime containing an extended object: a D-brane, an orientifold hyperplane, or a combination of the two. Is the reverse possible: starting from a theory with only closed oriented strings, can one obtain the other theories? This depends on a conjecture. Presumably, closed string field theory has a rich topological structure which leads to various extended objects as soliton solutions. We conjecture that the D-brane and the orientifold have such an interpretation. Our evidence for this is thin: since string theories are so resistant to coupling of external sources, we suspect that any object like the D-brane or orientifold which can couple consistently to closed strings is in fact made of closed strings. If this conjecture is correct, then one could start from a closed string state containing an appropriate soliton, compactify and obtain any of the other theories as a limit.

String theories can be distinguished by their world-sheet gauge algebra, (conformal or superconformal), by the world-sheet topologies allowed, and by their world-sheet Lagrangians. It is widely conjectured (see for example [12]) that theories with the same algebra and topology but different Lagrangians are in most or all cases different vacua of the same theory. The present work now provides evidence that theories with different world-sheet topologies are

in fact different vacua of a single theory. This leaves only the world-sheet gauge algebra to distinguish different string theories. We know of no reliable evidence that theories with different algebras can be connected, although the observation of L. Dixon (private communication) that Ramond-Ramond superstring backgrounds break superconformal invariance is suggestive.

We have found new extended objects, the bosonic D-brane and the orientifold plane, which can couple in a consistent way to closed oriented strings. This consistency follows at once, because these objects arise in dual descriptions of known consistent theories. On the other hand, the would-be super-D-brane, discussed in Section 5, is consistent in lowest order but in fact leads to anomalies.

String theories contain a rich spectrum of extended objects: the strings themselves, the D-branes, and the orbifold[13] and orientifold planes. These objects are all of zero thickness, to the extent that this has any meaning in string theory. Strings will also contain various soft extended objects, solitons in the low energy field theory. The zero thickness objects form an interesting hierarchy, distinguished by their tension. Let λ be the dimensionless coupling of three closed strings: λ is κ times an appropriate power of α'. We consider $1+1$ dimensional D-branes and orbifold/orientifold planes ($k = d-2$), to facilitate comparison with the string. Then the tension of the fundamental string is T, of the D-brane T/λ times a numerical constant (see (19)), and of the orbifold/orientifold T/λ^2 times a numerical constant. The last

identification is made by considering a $1+1$ dimensional orbifold plane in $3+1$ noncompact dimensions: the deficit angle is O(1), corresponding to a mass density $0(\kappa^{-2})$. In connection with our earlier conjecture, we note that T/λ^2 is what would be expected for a soliton made out of closed strings, while the value T/λ, the mean of the fundamental tension T and the soliton tension T/λ^2, suggests a self-dual point. Note also that the D-brane and orbifold/orientifold plane have arbitrary dimension, while the fundamental string is $1+1$ dimensional only. The reader is free to speculate about the role of fundamental p-branes.

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Footnotes

[f1] After we discovered this, but before completing the work in the following sections, we received the paper by Dine, Huet, and Seiberg [3], which reports the same result plus some additional connections involving twisted superstrings.

[f2] The numerical value $C = (\alpha')^6 2^{17} \pi^{23/2}$ is obtained in [7].

[f3] The dual coordinate might be more generally defined $Y^\mu = X_R^\mu + (-1)^{S^\mu} X_L^\mu + C^\mu$, for arbitrary constant C^μ, but this trivial redefinition changes nothing: both the D-brane and the orientifold plane are now at $Y^i = C^i$.

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